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TRAVELING WAVE WITH ALLOWANCE FOR EQUILIBRIUM RADIATION

S. P. Bautin and A. P. Sadov

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Three modes of propagation of a traveling-wave front over a noncold gas with different propagation velocities are found using one thermodynamic model. When the indicated velocity is low, transition from constant values of the gas parameters on both sides of the traveling-wave front proceeds continuously. An increase in the traveling-wave velocity leads to an isothermal jump: the density and velocity of the gas undergo a strong discontinuity whereas the temperature varies continuously. With a further increase in the traveling-wave velocity, the isothermal jump disappears and the flow becomes continuous again.

Key words: *radiative heat conduction, inviscid gas, traveling wave, isothermal jump.*

Obtaining high gas compression ratios is of great importance in solving many engineering problems (see, for example, [1]). In the case of great compression of gases, it is necessary to take into account equilibrium radiation [1–3]. In this case, the gas equations of state for the become

$$p = R\rho T + \sigma T^4/3, \qquad e = c_v T + \sigma T^4/\rho, \qquad \sigma = \text{const} > 0. \tag{1}$$

Here p is the pressure, ρ is the density, T is the temperature, e is the internal energy, c_v is the specific heat capacity at constant volume, R is the gas constant, σ is a constant related to the Stefan–Boltzmann constant σ_* by the formula $\sigma = 4\sigma_*/c_*$, where c_* is the velocity of light in vacuum.

A gas treated as a thermodynamic system is a two-parameter continuous medium [4, 5]. The gas density and temperature are considered independent thermodynamic variables. Therefore, the remaining thermodynamic parameters of the gas are functions of ρ and T, as for example, the pressure and internal energy given by relations (1).

If equilibrium radiation is taken into account in the equations of gas dynamics [4], the equation expressing the differential form of the energy conservation law becomes a nonlinear heat-conduction equation in a moving medium [5] and the heat conductivity becomes

$$\varkappa = 2\,\frac{\sigma c_* \alpha_*}{\gamma - 1}\,\frac{T^3}{\rho} \tag{2}$$

(α_* is a positive constant that depends on the choice of a system of units and $\gamma - 1 = R/c_v > 0$ is the polytropic exponent of an ideal gas).

Plane–parallel flows of a heat-conducting inviscid gas with the equations of state (1) and the heat conductivity (2) are described by the equations

$$\begin{gathered} \rho_t + u\rho_x + \rho u_x = 0, \\ u_t + uu_x + [T\rho_x + (\rho + \sigma_1 T^3)T_x]/(\gamma\rho) = 0, \\ (\rho + \sigma_2 T^3)(T_t + uT_x) + (\gamma - 1)T(\rho + \sigma_1 T^3)u_x = \varkappa_0\,\frac{\partial}{\partial x}\Big(\frac{T^3}{\rho}\,\frac{\partial T}{\partial x}\Big). \end{gathered} \tag{3}$$

Ural State University of Means of Communication, Ekaterinburg 620034; sbautin@math.usart.ru; alsadov@yandex.ru. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 47, No. 4, pp. 15–25, July–August, 2006. Original article submitted June 21, 2005; revision submitted August 30, 2005.

Here t is time, x is the spatial coordinate, u is the gas velocity,

$$\sigma_1 = \frac{4}{3}\,\sigma\,\frac{T_{00}^3}{R\rho_{00}}, \qquad \sigma_2 = 3(\gamma-1)\sigma_1, \qquad \varkappa_0 = \frac{2\sigma c_* \alpha_* T_{00}^3}{R u_{00} \rho_{00}^2 x_{00}},$$

and T_{00}, ρ_{00}, and x_{00} are the scale values of the temperature, density, and distance, respectively, for the variables made dimensionless in a standard manner. In this case, the scale value of the gas velocity is the sound velocity in a non-heat-conducting ideal gas: $u_{00} = \sqrt{R\gamma T_{00}}$.

The physical mechanism of radiative heat conduction differs from the mechanism of molecular heat conduction. In particular, in accounting for molecular heat conduction, it is necessary to take into account molecular viscosity because these effects are commensurable. This is suggested by the final value of the Prandtl number $\mathrm{Pr} = c_v \gamma \mu_{00}/\varkappa_{00}$, which is the ratio of the viscosity μ_{00} to the heat conductivity $\varkappa_{00}$ calculated for the scale values of ρ_{00} and T_{00}. For example, for air, it common to set $\mathrm{Pr} = 0.72$. In contrast to molecular heat conduction, the radiative heat conduction mechanism does not assume the occurrence of the viscosity effect in gas [1, 2, 6]. Therefore, system (3) does not contain terms due to the viscous interaction of gas particles. Formally, system (3) is obtained from the complete Navier–Stokes system (see, for example, [7]), in which one needs to take into account the particular types of the equations of state (1) and the heat conductivity (2) and to set the first and the second viscosity coefficients equal to zero.

If one sets $\varkappa_0 = 0$ in system (3), it becomes a hyperbolic system of gas-dynamic equations. Because ρ and T are chosen as independent thermodynamic variables and the energy equation is written for temperature, system (3) for $\varkappa_0 = 0$ differs in form from the traditional system of gas-dynamic equations [4] but, naturally, these systems are equivalent. The propagation velocity of the sound $C^{\pm}$-characteristics relative to the flow is determined by the sound velocity in a non-heat-conducting inviscid gas c. If $p = p(\rho, T)$ and $e = e(\rho, T)$, the sound velocity in a non-heat-conducting inviscid gas is specified by the relation

$$c = \sqrt{\frac{\partial p(\rho,T)}{\partial \rho} + \frac{T}{\rho^2}\left[\frac{\partial p(\rho,T)}{\partial T}\right]^2 \Big/ \frac{\partial e(\rho,T)}{\partial T}}. \tag{4}$$

In view of the equations of state (1), the formula specifying the sound velocity in the non-heat-conducting gas (i.e., for $\varkappa_0 = 0$) is written as

$$c = \sqrt{\frac{T}{\gamma}}\,\sqrt{1 + (\gamma-1)\,\frac{(1+\sigma_1 T^3/\rho)^2}{1+\sigma_2 T^3/\rho}}. \tag{5}$$

In the case of $\varkappa_0 \neq 0$, system (3) is of a mixed type: the first two equations form a hyperbolic part, and the last equation is parabolic. For this system, there are two sound $C_{\varkappa}^{\pm}$-characteristics [8] whose propagation velocity relative to the flow is equal to the sound velocity in the heat-conducting inviscid gas $c_{\varkappa}$:

$$c_{\varkappa} = \sqrt{\frac{\partial p(\rho,T)}{\partial \rho}}\,. \tag{6}$$

In the case of the equations of state (1), this quantity in dimensionless variables is given by the relation

$$c_{\varkappa} = \sqrt{T/\gamma}. \tag{7}$$

In the literature (see, for example, [2, 6, 9]) the quantity (6) is referred to as the isothermal sound velocity. We note, however, that the quantity $\sqrt{\partial p(\rho,T)/\partial \rho}$ determines the sound velocity in flows of heat-conducting inviscid gases irrespective of whether the flow temperature is constant or variable.

Because $\partial e(\rho,T)/\partial T > 0$ for ordinary gases, a comparison of formulas (4) and (6) leads to the inequality

$$c > c_{\varkappa}. \tag{8}$$

It is natural that inequality (8) is also satisfied in the particular case of the equations of state (1), where c and $c_{\varkappa}$ are specified by formulas (5) and (7), respectively.

Next, we consider the particular case of solutions of system (3): waves traveling over a noncold gas — solutions that depend on one independent variable

$$z = x - Dt, \qquad D = \mathrm{const} > 0; \tag{9}$$

in this case, in the region $z \to +\infty$, there is a homogeneous quiescent gas with the following gas-dynamic parameters:

$$\rho|_{z\to+\infty} = 1, \qquad u|_{z\to+\infty} = 0, \qquad T|_{z\to+\infty} = T_0 > 0. \tag{10}$$

The constant D specifies the velocity of the traveling wave that propagates from left to right.

Traveling waves in a heat-conducting ($\varkappa_0 \neq 0$) inviscid gas have been considered previously. For an ideal heat-conducting gas, for which $\sigma = 0$ in relations (1) but $\varkappa_0 \neq 0$, the cases $T_0 = 0$ (cold background) and $T_0 > 0$ have been considered. It has been shown [10] that flow with an isothermal jump occurs in an infinitely strong wave at $T_0 = 0$. At a certain point of this flow there is a discontinuity of the density, gas velocity, and heat flux but the temperature varies continuously over the entire flow [2, 5, 6, 9]. For the case $T_0 > 0$, it has been established [2, 5, 6] that a continuous transition occurs for small D and an isothermal jump for large D.

Traveling waves in a heat-conducting gas with the equations of state (1) (i.e., for $\sigma \neq 0$ and $\varkappa_0 \neq 0$) at $T_0 = 0$ are considered in [2, 9]. It has been shown that an isothermal jump occurs for small D and a continuous transition for large D, and, in addition, the following relation is valid:

$$\lim_{T_1\to+\infty} \frac{\rho_1}{\rho_0} = 7, \tag{11}$$

where ρ_1 and ρ_0 are the densities on the opposite sides of the infinitely strong traveling wave.

Unlike in the papers cited above, in the present paper, we study traveling waves at $T_0 > 0$ for the equations of state (1) with $\sigma \neq 0$ and $\varkappa_0 \neq 0$.

To analyze the properties of the waves traveling over a noncold background, we consider the system of ordinary differential equations

$$\begin{gathered} (u-D)\rho' + \rho u' = 0, \\ (u-D)u' + [T\rho' + (\rho + \sigma_1 T^3)T']/(\gamma\rho) = 0, \\ (\rho + \sigma_2 T^3)(u-D)T' + (\gamma-1)T(\rho+\sigma_1 T^3)u' = \varkappa_0 (T^3 T'/\rho)', \end{gathered} \tag{12}$$

which is obtained from system (3) using the substitution (9).

The first three integrals of system (12) are found in a standard way [5]:

$$\rho(u-D) = C_1; \tag{13}$$

$$T\rho + \sigma_1 T^4/4 = \gamma D u + C_2; \tag{14}$$

$$\varkappa_0 T^3 T'/\rho = \gamma(\gamma-1)Du^2/2 + (\gamma-1)C_2 u - DT - \sigma_2(D-u)T^4/4 + C_3. \tag{15}$$

Relations (13)–(15) are Hugoniot conditions for a heat-conducting inviscid gas [2, 5, 6], which can also be obtained from the integral conservation laws [5]. The left side of condition (15), which is one of the forms of the energy conservation law, contains the heat flux, which depends, in particular, on the temperature gradient, according to the Fourier law for heat conduction. If we set $\varkappa_0 = 0$ or $T' = 0$ in (15), the traditional Hugoniot conditions for a non-heat-conducting inviscid gas [4, 5] become.

The arbitrary constants C_1, C_2, and C_3 are unequally determined from conditions (10). Next for the sake of illustration, we set

$$T_0 = 1,$$

Therefore,

$$C_1 = -D, \qquad C_2 = 1 + \sigma_1/4, \qquad C_3 = D(1 + \sigma_2/4). \tag{16}$$

Integral (13) allows us to eliminate ρ from integral (14) and to obtain the relation

$$u_\pm(T) = [\gamma D^2 + \sigma_1 T^4/4 - (1 + \sigma_1/4) \pm \sqrt{W(T)}\,]/(2\gamma D), \tag{17}$$

where

$$W(T) = [\sigma_1 T^4/4 - (\gamma D^2 + 1 + \sigma_1/4)]^2 - 4\gamma D^2 T.$$

The function $W(T)$ has two positive roots T_* and T_{**} ($0 < T_* < T_{**}$), and it is strictly positive for $0 \leq T < T_*$ and $T > T_{**}$ and is strictly negative for $T_* < T < T_{**}$. Since the function $\sqrt{W(T)}$ is not defined in the range

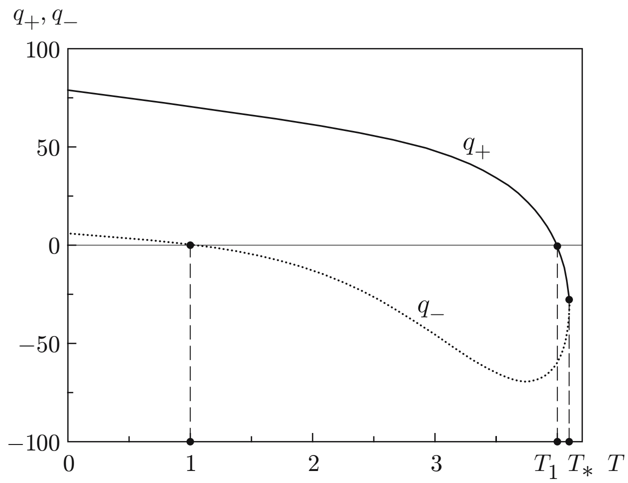

Fig. 1. Double-valued temperature dependence of the heat flux $q_\pm$.

$T_* < T < T_{**}$, it is necessary to examine only the interval $0 \le T \le T_*$. It is in this range that the double-valued function $u = u_\pm(T)$ is defined. The value of the derivative $du_\pm(T)/dx$ is calculated by the formula

$$\frac{du_\pm(T)}{dx} = \frac{du_\pm(T)}{dT}\,\frac{dT}{dx} = \frac{1}{2\gamma D}\Big[\sigma_1 T^3 \pm \frac{W'(T)}{2\sqrt{W(T)}}\Big]\,\frac{dT}{dx}.$$

Thus, at the point $T = T_*$, where $W(T_*) = 0$, the value of the derivative $du_\pm(T)/dx$ is obviously equal to infinity although the gas velocity at $T = T_*$ is finite.

From formulas (13) and (17), it follows that the dependence

$$\rho = \rho_\pm(T) \equiv D/(D - u_\pm(T)) \tag{18}$$

is defined in the half-interval $(0, T_*]$ and is also double-valued.

The obtained dependences (17) and (18) allow the gas density to be eliminated from the left side of relation (15) and the gas velocity from the right side of relation (15), which is denoted below as

$$q_\pm(T) = \gamma(\gamma-1)Du_\pm^2(T)/2 + (\gamma-1)(1+\sigma_1/4)u_\pm(T)$$

$$-DT - \sigma_2[D - u_\pm(T)]T^4/4 + D(1+\sigma_2/4). \tag{19}$$

In this case, $q_-(1) = 0$, $q_-(T_*) = q_+(T_*)$ and $q_-(T) < q_+(T)$ at $0 \le T < T_*$. As $z \to +\infty$, the limiting values of the gas density $\rho = 1$ and the gas velocity $u = 0$ belong to the lower branches of the corresponding curves $\rho_\pm(T)$ and $u_\pm(T)$ for $T = 1$; therefore, the zero value of the heat flux for $T = 1$ also belongs to the lower branch of the curve of $q_\pm(T)$. Figure 1 gives a curve of $q_\pm(T)$ for $D = 5$ and the following values of the constants: $\gamma = 5/3$, $\varkappa_0 = 1.4842$, and $\sigma_1 = 0.2366$. (The same values of the constants are used below.)

Let

$$D_0 = c(\rho, T)|_{\rho=T=1},$$

where $c(\rho, T)$ is calculated by formula (5). In the case considered, $D_0 \approx 1.00757$. For $D = D_0$ for the curve of $q_\pm(T)$, the value of $T = 1$ is a root of multiplicity two, which belongs to the lower branch: $q_-(1) = 0$, $q'_-(1) = 0$.

For $D > D_0$, the second root $T = T_1 > 1$ appears on the curve $q_\pm(T)$. It is first located on the lower branch $[q_-(T_1) = 0]$ but as D increases, it passes to the upper branch $[q_+(T_1) = 0]$, and then returns to the lower branch again. The location of the root $T = T_1$ on the different branches of the curve of $q_\pm(T)$ leads to substantially different gas flows: flow without a strong discontinuity or flow with a strong discontinuity.

Let the root $T = T_1 > 1$, as well as the root $T = 1$, be located on the lower branch: $q_-(T_1) = 0$. Then, the solution of the Cauchy problem for the ordinary differential equation

$$\varkappa_0 T^3 T'/\rho_-(T) = q_-(T), \qquad T|_{z=0} = T^0 \tag{20}$$

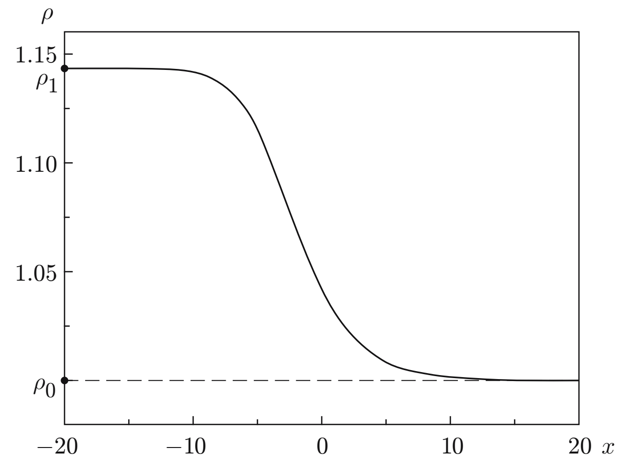

Fig. 2

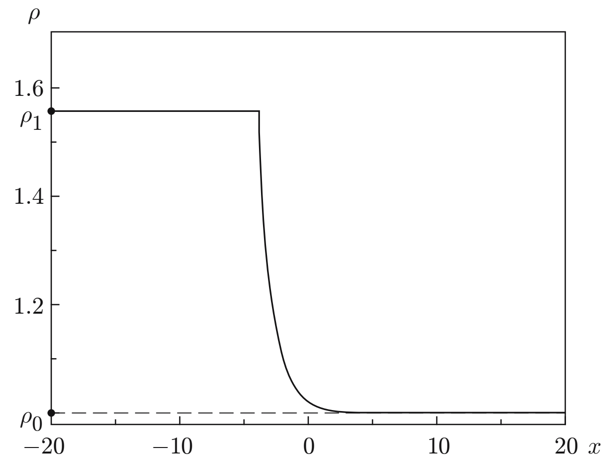

Fig. 3

Fig. 2. Continuous distribution of the gas density in the first mode.

Fig. 3. Continuous distribution of the gas density for transition from the first to the second mode.

yields the dependence $T = T(z)$ for $-\infty < z < +\infty$. Here the value of the constant T^0 should be chosen in the interval of 1 to T_1, for example, $T^0 = (1 + T_1)/2$.

Because the differential equation is cumbersome, the problem (20) is solved numerically. In this case, the limiting values $T|_{z=-\infty} = T_1$ and $T|_{z=+\infty} = 1$ are approached rather rapidly. For fixed values of the parameters γ, $\varkappa_0$, and σ_1, the rate of this approach depends on the constant D.

Once the dependence $T = T(z)$ is determined, the functions $u = u(z) \equiv u_-(T(z))$ and $\rho = \rho(z) \equiv \rho_-(T(z))$ are found from formulas (17) and (18) with the minus sign. Figure 2 shows the dependence $\rho(z)|_{t=0} = \rho(x)$ found by the indicated method for $D = 1.1$. The curves of $u(z)|_{t=0} = u(x)$ and $T(z)|_{t=0} = T(x)$ are qualitatively similar to the curve of $\rho(z)|_{t=0} = \rho(x)$. In this case, the traveling wave, as well as Becker's [5] solution, describes a monotonic shockless transition from the limiting values $\rho = \rho_1 \approx 1.1430$, $u = u_1 \approx 0.1376$, and $T = T_1 \approx 1.0776$ for $z = -\infty$ to the limiting values $\rho = 1$, $u = 0$, and $T = 1$ for $z = +\infty$.

Once a certain value $D = D_1 > D_0$ is reached, the root $T = T_1$ takes the boundary position, i.e., it belongs simultaneously to both branches of the curve of $q_\pm(T)$: $q_-(T_1) = q_+(T_1) = 0$. Naturally, this takes place only in the case where the root T_1 coincides with the boundary point of the range of definition of the functions $q_\pm(T)$, $\rho_\pm(T)$, and $u_\pm(T)$: $T_1 = T_*$. Since the root T_1 is on the lower branch, the transition from the values $\rho_0 = 1$, $u_0 = 0$, and $T_0 = 1$ to the values ρ_1, u_1, and T_1 occurs continuously but for infinite values of the derivatives of the gas density and velocity with respect to the variable z on the left boundary of the smeared shock wave. In this case, since $q_-(T_1) = 0$, the derivative $T'(z)$ is continuous and on the left boundary of the smeared shock wave, it is equal to zero. For the version used, the following values of the constants are obtained: $D_1 \approx 1.366$, $T_* \approx 1.289702$, $\rho_1 = \rho_{-\infty} \approx 1.5520$, $u_1 = u_{-\infty} \approx 0.4859$, and $T_1 = T_{-\infty} \approx 1.2897$. The behavior of the curve of $\rho|_{t=0} = \rho(x)$ for this case is given in Fig. 3.

With a further increase in D, the root $T = T_1$ passes to the upper branch of the curve of $q_\pm(T)$: $q_+(T_1) = 0$. In this case, as z decreases, the function $T(z)$ takes the value $T = T_1$ for a certain finite value $z = z_1$. At the point $z = z_1$, the quantities q, ρ and u must satisfy the strong discontinuity condition — transition from the values $q = q_2 = q_-(T_1)$, $\rho = \rho_2 = \rho_-(T_1)$, and $u = u_2 = u_-(T_1)$ to the values $q = q_1 = q_+(T_1)$, $\rho = \rho_1 = \rho_+(T_1)$, and $u = u_1 = u_+(T_1)$, respectively. For this discontinuous transition from the lower to the upper branches, all conservation laws (13)–(15) are satisfied since the dependences $\rho_\pm(T)$, $u_\pm(T)$, $q_\pm(T)$ are defined for the same set (16) of constants C_1, C_2, and C_3.

Since $q_+(T_1) = 0$, the solution of the Cauchy problem

$$\varkappa_0 T^3 T'/\rho_+(T) = q_+(T), \qquad T|_{z=z_1} = T_1$$

for $-\infty < z < z_1$ is the constant temperature value $T = T_1$, which leads to the constant values of the gas parameters:

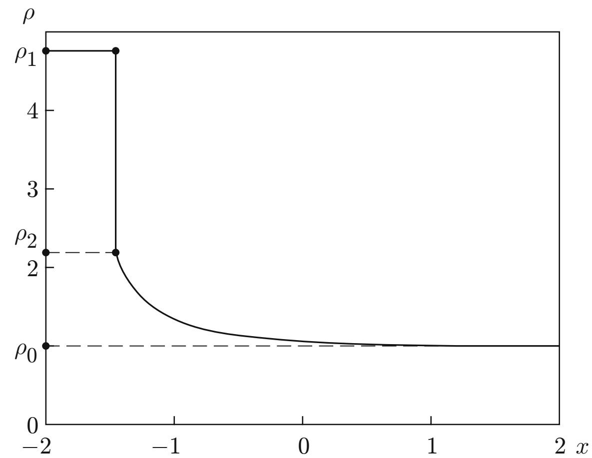

Fig. 4. Isothermal density jump in the second mode.

$\rho = \rho_1 > 1$, $u = u_1 > 0$, and $T = T_1 > 1$. Figure 4 gives the dependence $\rho|_{t=0} = \rho(x)$ for the values of the constants γ, $\varkappa_0$, σ_1, and $D = 5$. In this version, $\rho_1 \approx 4.7416 > \rho_2 \approx 2.2016 > \rho_0 = 1$, $u_1 \approx 3.9455 > u_2 \approx 2.7289 > u_0 = 0$, $T_1 \approx 3.9915 > T_0 = 1$, and $T_* \approx 4.0934$. In this case, the temperature varies continuously: $T(z) = T_1$ for $z \le z_1$; for $z \ge z_1$, the function $T(z)$ decreases monotonically from T_1 to unity with increasing z. The dependence $T(z)$ behaves the same as $\rho(z)$ for $D = 1.366$ (see Fig. 3): at the point z_1, the derivative $T'(z)$ has a discontinuity but there is no infinite gradient.

The flow ahead of the shock transition considered is called a thermal precursor. In the case where the temperature of the gas in rest is equal to zero (for $z = +\infty$), the traveling wave becomes a wave that propagates at a finite velocity over the cold gas [1, 2, 9, 10]. Then, the width of the thermal precursor is finite: from $z = z_1$ to $z = z_0 > z_1$, where z_0 is the coordinate of the thermal-wave front.

In the present paper, we consider the version $T|_{z\to+\infty} > 0$. Therefore, in the cases studied, the thermal heterogeneity propagates at an infinite velocity and the width of the thermal precursor is infinite: from $z = z_1$ to $z = +\infty$. However, as noted above, the approach of the gas parameters in the thermal precursor to the limiting values for $z \to \pm\infty$ is rather fast (see Figs. 2–4).

For $D > D_1$, the root $T = T_1$ again passes from the branch $q_+(T)$ to the branch $q_-(T)$ and there is a boundary situation: for a certain $D = D_2 > D_1$, the root $T = T_1$ coincides with the boundary point of the range of definition of the curve $q_\pm(T)$ and belongs simultaneously to both branches: $q_+(T_*) = q_-(T_*) = 0$. In this case, as well as for $D = D_1$, a strong discontinuity in the gas flow is absent. On the left boundary of the smeared shock wave, the derivatives of the density and velocity with respect to the space variable are infinite and the derivative $T'(z)$ is continuous and vanishes at the point $T_* = 0$. For the values of the constants used in the calculations, $D_1 \approx 13.675$. The curve $\rho|_{t=0} = \rho(x)$ in this case behaves the same as that in Fig. 3 but the value of ρ_1 is larger.

With a further increase in D ($D > D_2$), a strong discontinuity disappears and the traveling wave, as in the case of $D_0 < D < D_1$, transfers the smeared shock transition with the values of the gas parameters for $z = -\infty$.

Thus, in the case $\sigma \ne 0$, $\varkappa_0 \ne 0$, $T_0 > 0$, there are three traveling wave modes: a continuous transition for $D_0 < D \le D_1$, an isothermal jump for $D_1 < D < D_2$, and a continuous transition for $D_2 \le D$.

The values of the gas parameters on the different sides of the traveling wave

$$\boldsymbol{U}_0 = \boldsymbol{U}|_{z\to+\infty}, \qquad \boldsymbol{U}_1 = \boldsymbol{U}|_{z\to-\infty}, \qquad \boldsymbol{U} = (\rho, u, T)$$

are related by the Hugoniot conditions (13)–(15); since in the examined flows, $\lim\limits_{z\to\pm\infty} T' = 0$, these conditions coincide with the Hugoniot conditions for a non-heat-conducting inviscid gas [4]. Therefore, in particular, the determinateness theorem [4] is valid. According to this theorem, the values of $\boldsymbol{U}_1$ are uniquely determined from the values $\boldsymbol{U}_0$ and D irrespective of whether in the heat-conducting gas flow there an isothermal jump or not. Because an isothermal jump is absent for $D_0 < D < D_1$ and at $D_2 < D$, the gas-dynamic parameters in the examined traveling waves for these values of D change continuously and monotonically from the values $\boldsymbol{U}_1 = \boldsymbol{U}|_{z\to-\infty}$ to the values $\boldsymbol{U}_0 = \boldsymbol{U}|_{z\to+\infty}$, which are related by the Hugoniot conditions for a non-heat-conducting inviscid gas.

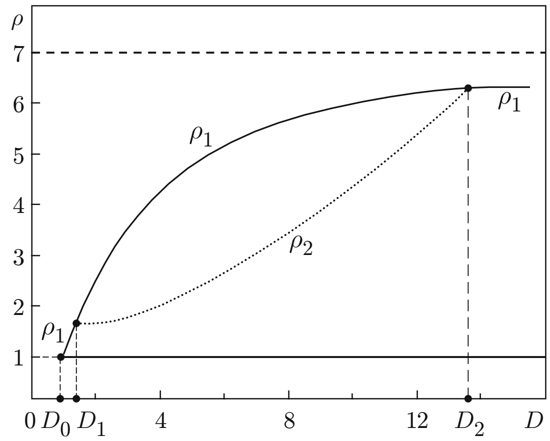

Fig. 5

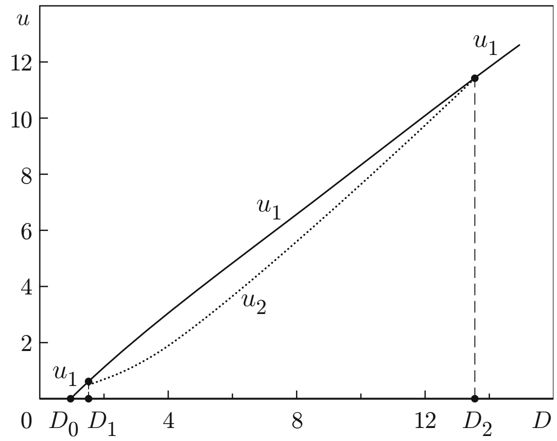

Fig. 6

Fig. 5. Gas density on the different sides of the traveling-wave front ($\rho_{+\infty} = 1$ and $\rho_{-\infty} = \rho_1$) and in the isothermal jump (ρ_2–ρ_1); $\rho = 7$ is the limiting value of the density change in the wave traveling over a heat-conducting inviscid gas.

Fig. 6. Gas velocity on the different sides of the traveling-wave front ($u_{+\infty} = 0$ and $u_{-\infty} = u_1$) and in the isothermal jump (u_2–u_1).

Because for $D_1 < D < D_2$, there is an isothermal jump in the gas flow, it follows, first, that the temperature for $z \le z_1$ is constant ($T = T_1$), and for $z \ge z_1$, the temperature varies continuously and monotonically from the value $T_1 = T|_{z=z_1}$ to the value $T_0 = T|_{z\to+\infty}$. Second, the gas density and velocity behave differently: in the thermal precursor with decreasing z (from $+\infty$ up to z_1), they change monotonically and continuously from the values $\rho_0 = \rho|_{z\to+\infty}$ and $u_0 = 0 = u|_{z\to+\infty}$ to the values $\rho_2 = \rho_-(T_1) > \rho_0$ and $u_2 = u_-(T_1) > 0$, respectively, at the point $z = z_1$; then, at the point $z = z_1$, there is a sudden transition to the values $\rho_1 = \rho_+(T_1) > \rho_2$ and $u_1 = u_+(T_1) > u_2$, respectively. Next, on the entire semiaxis $(-\infty, z_1]$, the gas density and velocity are constant: $\rho = \rho_1$ and $u = u_1$. For the chosen values of the constants γ, $\varkappa_0$, and σ_1, curves of $\rho_1(D)$ and $\rho_2(D)$ are given in Fig. 5, and curves of $u_1(D)$ and $u_2(D)$ in Fig. 6.

For a non-heat-conducting inviscid gas, the Zemplén theorem [4, 5] for shock waves is valid:

$$|u_0 - D| > c_0, \qquad |u_1 - D| < c_1. \tag{21}$$

According to this theorem, the shock-wave front catches up with the weak perturbations that arise ahead of it and the weak perturbations available behind the shock-wave front, in turn, catch up with it. In the literature, this property is sometimes referred to as the evolutionarity property [6].

Naturally, if for the specified values of $\boldsymbol{U}_0$, $\boldsymbol{U}_1$ and D, the sound velocity is calculated by formulas (4) and (5) (i.e., in the case of a non-heat-conducting gas with $\varkappa_0 = 0$), the Zemplén theorem holds. However, if the sound velocity is calculated by formulas (6) and (7), which is valid for heat-conducting inviscid gas flows, it is not known *a priori* whether such flows possesses the evolutionarity property. Moreover, direct calculations of the quantities (21) using the sound velocity in a heat-conducting inviscid gas as the sound velocity give a peculiar picture of the feasibility of the evolutionarity property for the waves traveling over the noncold background in a heat-conducting inviscid gas.

For the flow ahead of the traveling-wave front with allowance for the value $u_0 = 0$ the evolutionarity property is written as

$$D > c_\varkappa(\rho_0, T_0)$$

and holds. Indeed, under the Zemplén theorem, $D > c(\rho_0, T_0)$. Then, in view of relation (8), we obtain the chain of inequalities

$$D > c(\rho_0, T_0) > c_\varkappa(\rho_0, T_0),$$

which implies the evolutionarity of the flow ahead of the traveling-wave front. It should be noted that the need to satisfy the Zemplén theorem in the case of a non-heat-conducting gas [$D > c(\rho_0, T_0)$] determines the value $D_0 = c(\rho_0, T_0)$.

For the heat-conducting inviscid gas flow behind the traveling-wave front, calculations for the employed values of the parameters γ, $\varkappa_0$, and σ_1 show the following. If in the gas flow there is an isothermal jump (i.e., $D_1 < D < D_2$), then

$$c_\varkappa(\rho_1, T_1) > D - u_1,$$

i.e., the Zemplén theorem is satisfied and the evolutionarity property holds. However, if the heat-conducting inviscid gas flow in the traveling wave is continuous ($D_0 < D < D_1$ and $D_2 < D$), the following inequality is valid:

$$c_\varkappa(\rho_1, T_1) < D - u_1; \tag{22}$$

consequently, weak discontinuities that for some reasons arise behind the traveling wave front may not catch up with the wave front.

The fact is that system (3) takes into account two mechanisms of perturbation transfer: elastic interaction and radiative heat conduction. As is known, in the case of a background with nonzero temperature, the thermal heterogeneity propagates at an infinite velocity. Exactly this circumstance is responsible for the infinite width of the thermal precursor ahead of the traveling wave front (see Figs. 2–4). Consequently, the presence of the thermal conduction mechanism causes heating of the gas and a rise in the temperature, which, in turn, leads to an increase in the sound velocity in the heat-conducting inviscid gas. Therefore, generally speaking, in spite of the validity of inequality (22), weak perturbations can catch up with the traveling wave front.

However, there are examples of flows (see [8, 11]) where weak perturbations produced by a smooth displacement of an impermeable piston in a homogeneous gas of nonzero temperature propagate over a homogeneous background at a constant velocity $c_\varkappa$ and do not lead to an infinite propagation velocity of the thermal heterogeneity. This effect arises when on the compressing piston there is a heat drain under a special law. These examples suggest that for $D_0 < D < D_1$ and $D_2 < D$, the evolutionarity property in heat-conducting inviscid gas flows may not be satisfied.

Next, we consider the influence of T_0 on the gas flow. As noted above, the value of $T_0 = 1$ is chosen only for a better visualization of calculation results. The values of the constants for $T_0 = 1$ [2, 12] used in the calculations correspond to a temperature of about 1 keV, i.e., 10^7 K. Obviously, in laser thermonuclear fusion experiments, this temperature is attained not on the target in the initial state but in the already compressed gas heated to a very high temperature.

Calculations for values of T_0 decreasing to $T_0 = 10^{-6}$ K showed that for $T_0 > 0$ there are three modes of traveling-wave propagation: a continuous flow for $D_0(T_0) < D < D_1(T_0)$ and $D_2(T_0) < D$ and an isothermal jump for $D_1(T_0) < D < D_2(T_0)$. In this case, if the value of T_0 decreases to zero, then $D_{0,1}(T_0) \to 0$ and

$$D_2(T_0) \to D_{2*} = \Big[\frac{(8+4s)(4+s)^7}{\sigma_1 \gamma^3}\Big]^{1/6}, \qquad s = \sqrt{\frac{3\gamma - 1}{\gamma - 1}}.$$

Consequently, in the limit as $T_0 \to 0$, the three modes continuously become two modes for the waves traveling over a cold background [2, 9]. This occurs because the range of values D for which the first continuous-flow mode occurs ($D_0 < D < D_1$) decreases to zero as $T_0 \to 0$.

In all calculated versions for $T_0 > 0$, the evolutionarity property is manifested up in the same manner as in the case $T_0 = 1$: for the flow ahead of the wave front, and for the flow behind the front, but inequality (21) with $c_\varkappa$ taken as the sound velocity holds only in the presence of an isothermal jump. In the cases of continuous profiles of gas-dynamic parameters in the traveling wave, inequality (22) holds.

The literature (see, for example, [2]) gives values of the gas parameters (temperature, transparency boundary, etc.,) for which thermal radiation becomes significant for both the flow ahead of the shock transition and the flow at and behind the shock-wave front. It seems that the solutions constructed and the values of the constants introduced (in particular, for $T_0 \to 0$) allow the validity of using the radiative heat conduction approximation to be determined for different versions. In addition, in using the equilibrium radiation model, it is necessary to take into account that for $\sigma \to 0$, the limiting transition in relations (1)–(3) does not lead to the corresponding transition of the solutions with $\sigma \neq 0$ to the solutions with $\sigma = 0$ [see, for example, formulas (5) and (7)].

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